

# Direct dark matter detection: The diurnal variation in directional experiments

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## Abstract

We present some theoretical results relevant to the direct dark matter detection experiments, paying particular attention to directional experiments, i.e. experiments in which, not only the energy but the direction of the recoiling nucleus is observed. In directional experiments the detection rate depends on the angle between the line observation and the sun's direction of motion. Since, however, the direction of observation is fixed with respect the earth, while the Earth is rotating around its axis, in a directional experiment the angle between the direction of observation and the Sun's direction of motion will change during the day. So the observed signal in such experiments will exhibit a very interesting and characteristic periodic diurnal variation.

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## 1 Introduction

The combined MAXIMA-1 [1], BOOMERANG [2], DASI [3] and COBE/DMR Cosmic Microwave Background (CMB) observations [4] imply that the Universe is flat [5] and that most of the matter in the Universe is Dark [6], i.e. exotic. These results have been confirmed and improved by the recent WMAP data [7]. Combining the the data of these quite precise experiments one finds:

$$\Omega_b = 0.0456 \pm 0.0015, \quad \Omega_{CDM} = 0.228 \pm 0.013, \quad \Omega_\Lambda = 0.726 \pm 0.015.$$

Since any "invisible" non exotic component cannot possibly exceed 40% of the above  $\Omega_{CDM}$  [8], exotic (non baryonic) matter is required and there is room for cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles).

Since the WIMP's are expected to be very massive and extremely non relativistic, they are not likely to excite the nucleus. So they can be directly detected mainly via the recoiling of a nucleus (A,Z) in elastic scattering. In the standard nuclear recoil experiments, first proposed more than 30 years ago [9], one has to face the problem that the reaction of interest does not have a characteristic feature to distinguish it from the background. So for the expected low counting rates the background is a formidable problem. Some special features of the WIMP-nuclear interaction can be exploited to reduce the background problems. Such are: i) the modulation effect: this yields a periodic signal due to the motion of the earth around the sun. ii) backward-forward asymmetry expected in directional experiments, i.e. experiments in which the direction of the recoiling nucleus is also observed. iii) transitions to very low excited states, if present in the target: in this case one need not measure nuclear recoils, but the de-excitation  $\gamma$  rays. (iv) detection of electrons produced during the WIMP-nucleus collision or subsequent X-rays.

In our previous work [10, 11, 12] we have found that the observed directional rate is characterized by a strong dependence on the angle between the direction of nuclear recoil and the direction of the sun's motion, with respect to the center of the galaxy. The apparatus will, of course, be oriented in a direction defined in the local frame, i.e. the line of observation will point to a point in the sky specified, in the equatorial system, by right ascension  $\tilde{\alpha}$  and inclination  $\tilde{\delta}$ . Thus due to the rotation of the earth in a directional experiment the angle between the direction of observation and the Sun's direction of motion will change during the day. So, since the event rates sensitively depend on this angle, as we have recently suggested [13] one may be able to observe a diurnal variation. In the present paper we will explore this novel feature of directional experiments, i.e. the variation of the data with a period of

24 hours. This variation is independent of the Earth's rotational velocity. Its amplitude depends, in addition to some of the usual parameters specifying the standard WIMP event rate, on the inclination of observation, i.e. the angle between the direction of observation and the axis of the Earth's rotation. Those features cannot be masked by any known background.

## 2 Standard (non Directional) Rates

The event rate for a given WIMP velocity  $v$  is given by [14]

$$\frac{dR}{du} = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sqrt{\langle v^2 \rangle} A^2 \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \frac{dt(u, v)}{du}, \quad (1)$$

where  $m_t/Am_p$  is the number of nuclei in a target of mass  $m_t$ ,  $A$  is the mass number of the target,  $u$  is the energy transfer in some convenient units,  $u = Q/Q_0$ ,  $Q_0 = 40A^{-4/3}$  MeV, and

$$\frac{dt(u, v)}{du} = \frac{v}{\sqrt{\langle v^2 \rangle}} F^2(u) \frac{1}{2(\mu_r bv)^2}, \quad (2)$$

with  $F(u)$  the nuclear form factor. We must now fold  $dt(u, v)/du$  with the velocity distribution  $f(y, \xi)$  in the local frame. Here we use  $y = v/v_0$ ,  $v_0$  is the sun's velocity with respect to the center of the galaxy, and  $\xi$  is the cosine of the the WIMP velocity with the polar axis (chosen to be the sun's direction of motion). We will assume in this work that the velocity distribution is Maxwell-Boltzmann in the galactic frame, namely

$$f(\mathbf{y}') = \frac{1}{\pi\sqrt{\pi}} e^{-(\mathbf{y}')^2}, \quad \mathbf{y}' = \frac{v'}{v_0}, \quad (3)$$

where  $v'$  is the WIMP velocity in the galactic frame. Obviously to obtain the distribution in the local frame one must take into account the motion of the Sun and Earth:

$$\mathbf{y}' = \mathbf{y} + \hat{z} + \delta (\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z}), \quad (4)$$

where  $\delta = v_1/v_0 = 0.136$  with  $v_1$  the velocity of the earth around the sun,  $\alpha$  the phase of the earth ( $\alpha=0$  on June 3rd) and  $\gamma \approx \pi/6$ .

In the case where  $\delta = 0$ , we get for the differential (with respect to the energy transfer) rate

$$\frac{dR}{du} = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{\langle v^2 \rangle} A^2 \frac{dt}{du}, \quad \frac{dt}{du} = \sqrt{\frac{2}{3}} a^2 F^2(u) \Psi_0(a\sqrt{u}), \quad (5)$$

$$\Psi_0(a\sqrt{u}) = \int_{a\sqrt{u}}^{y_{esc}} y dy \, 2\pi \int_{-1}^1 d\xi f(y, \xi), \quad (6)$$

where

$$a = (\sqrt{2}\mu_r bv_0)^{-1}. \quad (7)$$

We note that the function  $\Psi_0(x)$ , a decreasing function of the energy transfer, depends only on the velocity distribution and is independent of nuclear physics. An additional suppression as the energy transfer increases comes, of course, from the nuclear form factor  $F(u)$ .

Note that the nuclear dependence of the differential rate comes mainly from the form factor and, to some extent, via the parameter  $a$ . Integrating the differential event rate from  $u_{min} = E_{th}/Q_0$ , which depends on the energy threshold, to  $u_{max} = y_{esc}^2/a^2$  (where  $y_{esc} = 2.84$  since  $v_{esc} = 2.84v_0$ ) we obtain the total rate

$$R = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{\langle v^2 \rangle} A^2 t, \quad t = \int_{u_{min}}^{u_{max}} du \frac{dt}{du}. \quad (8)$$

In the case where  $\delta \neq 0$  we proceed as above and finally get for the differential rate an expression similar to (5) where now one has to replace:

$$\Psi_0(x) \rightarrow \Psi_0(x) + \Psi_1(x, \gamma, \delta) \cos \alpha, \quad (9)$$

where  $\Psi_1(x, \gamma, \delta)$  is obtained as the linear term in an expansion of the velocity distribution in powers of  $\delta$ . Thus one has to replace:

$$\frac{dt}{du} \rightarrow \frac{dr}{du} = \frac{dt}{du} + \frac{d\tilde{h}}{du} \cos \alpha, \quad (10)$$

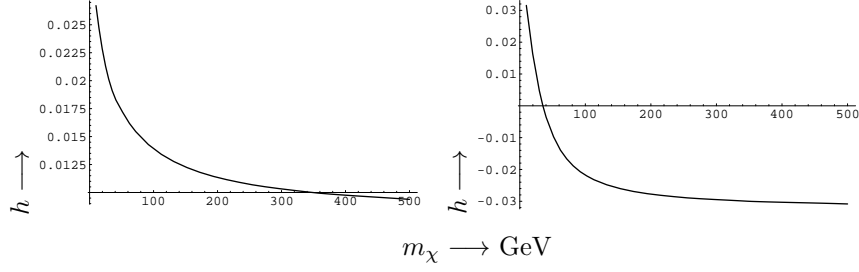


Figure 1: The quantity  $h$ , entering in the coherent mode discussed in this work, is shown as a function of the WIMP mass in GeV for  $Q_{min} = 0$  for a light system,  $^{32}\text{S}$  (left) and  $^{127}\text{I}$  or  $^{131}\text{Xe}$  (right).

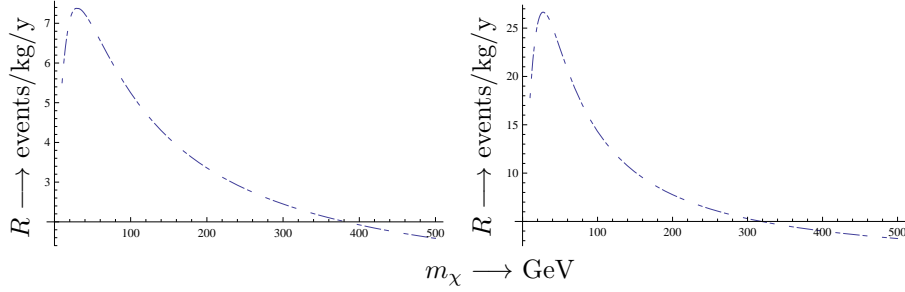


Figure 2: The coherent total event rate, as a function of the WIMP mass in GeV, is exhibited assuming a nucleon cross section  $\sigma_n = 10^{-43} \text{ cm}^2 = 10^{-7} \text{ pb}$  for  $Q_{min} = 0$  for a light system,  $^{32}\text{S}$  (left) and a heavy system,  $^{131}\text{Xe}$  (right).

$$t \rightarrow t(1 + h \cos \alpha), \quad h = \frac{1}{t} \int_{u_{min}}^{u_{max}} du \frac{d\tilde{h}}{du}. \quad (11)$$

Thus we finally get

$$R = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{\langle v^2 \rangle} A^2 t (1 + h \cos \alpha). \quad (12)$$

### 3 Directional Rates

In this instance the experiments will attempt to measure not only the energy, but the direction of the recoiling nucleus as well [15, 11, 12, 14]. Let us indicate the direction of observation by:

$$\hat{e} = (e_x, e_y, e_z) = (\sin \Theta \cos \Phi, \sin \Theta \sin \Phi, \cos \Theta). \quad (13)$$

Now Eq. (2) must be motified since no integration over the azimuthal angle specifying the direction of the outgoing nucleus is performed. Thus Eqs. (1) and (2) now become

$$\left( \frac{dR}{du} \right)_{dir} = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sqrt{\langle v^2 \rangle} A^2 \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \frac{1}{2\pi} \left( \frac{dt(u, v)}{du} \right)_{dir}, \quad (14)$$

$$\left( \frac{dt(u, v)}{du} \right)_{dir} = \frac{v}{\sqrt{\langle v^2 \rangle}} F^2(u) \frac{1}{2(\mu_r b v)^2} \delta \left( \hat{v} \cdot \hat{e} - \frac{a\sqrt{u}}{v/v_0} \right). \quad (15)$$

The factor  $1/(2\pi)$  enters since we are going to make use of the same nuclear cross section as in the non directional case.

The total event rate is obtained after folding with the velocity distribution and integrating the above expression over the energy transfer. It can be cast in the form:

$$R_{dir} = \frac{\rho_\chi}{m_\chi} \frac{m_t}{Am_p} \sigma_n \left( \frac{\mu_r}{\mu_p} \right)^2 \sqrt{\langle v^2 \rangle} A^2 \frac{1}{2\pi} t_{dir}(\Theta, \Phi) \left( 1 + h_c(\Theta, \Phi) \cos \alpha + h_s(\Theta, \Phi) \sin \alpha \right). \quad (16)$$

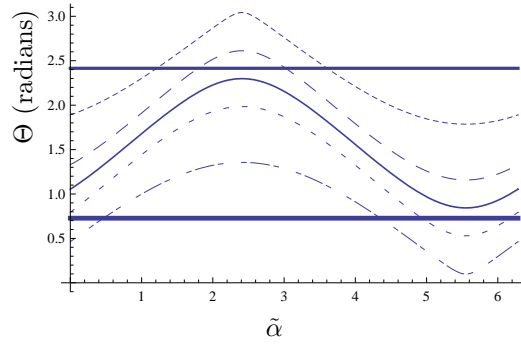


Figure 3: Due to the diurnal motion of the Earth the angle  $\Theta$ , between the line of observation and the sun's direction of motion, is changing as the earth rotates. We show the angle  $\Theta$  as a function of  $\tilde{\alpha}$ , essentially time, for various inclinations  $\tilde{\delta}$ . The intermediate thickness, the short dash, the long dash, the fine line, the long-short dash, the short-long-short dash and the thick line correspond to inclination  $\tilde{\delta} = -\pi/2, -3\pi/10, -\pi/10, 0, \pi/10, 3\pi/10$  and  $\pi/2$  respectively. We see that, for negative inclinations, the angle  $\Theta$  can take values near  $\pi$ , i.e. opposite to the direction of the sun's velocity, where the rate attains its maximum.

We found it convenient to use a new relative parameter  $\kappa^1$  as well as the parameters  $h_m$  and  $\alpha_0$  given by:

$$\kappa(\Theta, \Phi) = \frac{t_{dir}}{t}, \quad h_m(\Theta, \Phi) \cos(\alpha + \alpha_0) = h_c(\Theta, \Phi) \cos \alpha + h_s(\Theta, \Phi) \sin \alpha. \quad (17)$$

These parameters depend, of course, on the reduced mass  $\mu_r$ , the WIMP velocity distribution and, to some extent, on the nuclear physics via the nuclear form factor. The parameter  $\kappa$  gives the retardation factor of the directional rate, over and above the factor of  $1/(2\pi)$ , compared to the standard rate. Since, however, the unmodulated amplitude is independent of  $\Phi$  one can integrate over  $\Phi$  so that the suppression of  $1/(2\pi)$  drops out for this term. Because of the existence of both  $\cos \alpha$  and  $\sin \alpha$  terms the time dependence will be of the form  $\cos(\alpha + \alpha_0)$ , with the phase  $\alpha_0$  being direction dependent. Thus the time of the maximum and minimum will depend on the direction. In other words the seasonal dependence will depend on the direction of observation. So it cannot be masked by irrelevant seasonal effects.

Before concluding this section we like to consider the case connected with the partly directional experiments, i.e. experiments which can determine the line along which the nucleus is recoiling, but not the sense of direction on it. The results in this case can be obtained by summing up the events in both directions. i.e. those specified by  $(\Theta, \Phi)$  as well as  $(\pi - \Theta, \Phi + \pi)$ . A given line of observation is now specified by  $\Theta, \Phi$  in the range:

$$0 \leq \Theta \leq \pi/2, \quad 0 \leq \Phi \leq \pi.$$

The parameter  $h$  entering the non directional case is shown in Fig. 1. Using the above parameters  $t$  and employing Eq. (12) we obtain the time independent total event rate. The obtained results are shown in Fig. 2. The total modulation, since it is defined relative to the time independent part, is still given by Fig. 1.

## 4 Diurnal Variation

Up to now we have considered the event rate in a directional experiment in fixed direction with respect to the galaxy in the galactic system discussed above. The apparatus, of course, will be oriented in a direction specified in the local frame, e.g. by a point in the sky specified, in the equatorial system, by right ascension  $\tilde{\alpha}$  and inclination<sup>2</sup>  $\tilde{\delta}$ . This will lead to a diurnal variation of the event rate [13, 14].

The galactic frame, in the so called J2000 system, is defined by the galactic pole with ascension  $\tilde{\alpha}_1 = 12^h 51^m 26.282^s$  and inclination  $\tilde{\delta}_1 = +27^\circ 7' 42.01''$  and the galactic center at  $\tilde{\alpha}_2 = 17^h 45^m 37.224^s$ ,  $\tilde{\delta}_2 = -(28^\circ 56' 10.23'')$ . Thus the galactic unit vector  $\hat{y}$ , specified by  $(\tilde{\alpha}_1, \tilde{\delta}_1)$ , and the unit vector  $\hat{s}$ , specified

<sup>1</sup>This is done to avoid or minimize, e.g., the dependence on the parameters of the particle model and, in particular, the unknown WIMP-nucleon cross section. Furthermore the directional experiments can also obtain the rate in all directions. Thus  $\kappa$  maybe of experimental interest.

<sup>2</sup>We have chosen to adopt the notation  $\tilde{\alpha}$  and  $\tilde{\delta}$  instead of the standard notation  $\alpha$  and  $\delta$  employed by the astronomers to avoid possible confusion stemming from the fact that  $\alpha$  has already been used to designate the phase of the Earth and  $\delta$  has been used for the ratio of the rotational velocity of the Earth around the Sun by the velocity of the sun around the center of the galaxy

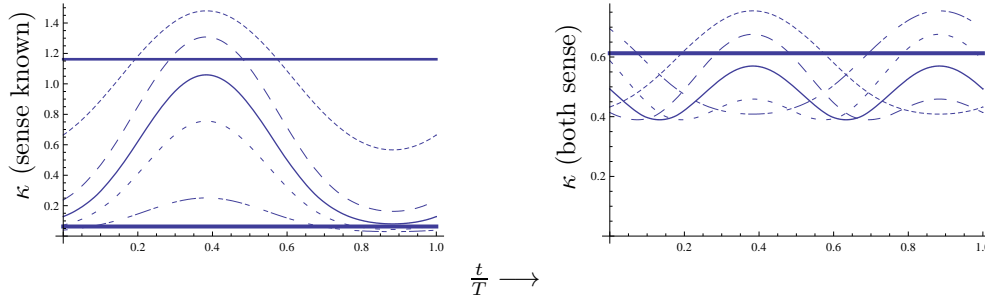


Figure 4: The time dependence (in units of the Earth's rotation period) of the parameter  $\kappa$  for various inclinations  $\tilde{\delta}$  in the case of the target  $\text{CS}_2$ . The intermediate thickness, the short dash, the long dash, the fine line, the long-short dash, the short-long-short dash and the thick line correspond to inclination  $\tilde{\delta} = -\pi/2, -3\pi/10, -\pi/10, 0, \pi/10, 3\pi/10$  and  $\pi/2$  respectively. Due to the diurnal motion of the Earth different angles  $\Theta$  are sampled as the earth rotates.

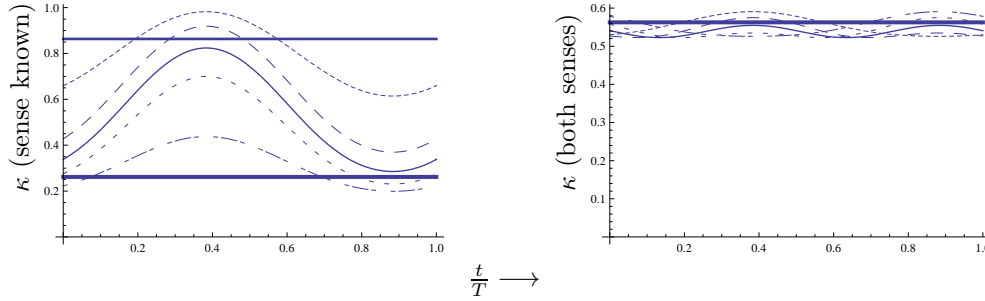


Figure 5: The same as in Fig. 4 in the case of the Iodine target. The intermediate thickness, the short dash, the long dash, the fine line, the long-short dash, the short-long-short dash and the thick line correspond to inclination  $\tilde{\delta} = -\pi/2, -3\pi/10, -\pi/10, 0, \pi/10, 3\pi/10$  and  $\pi/2$  respectively.

by  $(\tilde{\alpha}_2, \tilde{\delta}_2)$ , can be expressed in terms of the celestial unit vectors  $\hat{i}$  (beginning of measuring the right ascension),  $\hat{k}$  (the axis of the Earth's rotation) and  $\hat{j} = \hat{k} \times \hat{i}$ . One finds

$$\begin{aligned}\hat{y} &= -0.868\hat{i} - 0.198\hat{j} + 0.456\hat{k} \text{ (galactic axis) ,} \\ \hat{x} &= -\hat{s} = 0.055\hat{i} + 0.873\hat{j} + 0.483\hat{k} \text{ (radially out towards the sun) ,} \\ \hat{z} &= \hat{x} \times \hat{y} = 0.494\hat{i} - 0.445\hat{j} + 0.747\hat{k} \text{ (the sun's direction of motion).}\end{aligned}\tag{18}$$

Note in our system the x-axis is opposite to the s-axis used by the astronomers. Thus a vector oriented by  $(\tilde{\alpha}, \tilde{\delta})$  in the laboratory is given in the galactic frame by a unit vector with components:

$$\begin{pmatrix} y \\ x \\ z \end{pmatrix} = \begin{pmatrix} -0.868 \cos \tilde{\alpha} \cos \tilde{\delta} - 0.198 \sin \tilde{\alpha} \cos \tilde{\delta} + 0.456 \sin \tilde{\delta} \\ 0.055 \cos \tilde{\alpha} \cos \tilde{\delta} + 0.873 \sin \tilde{\alpha} \cos \tilde{\delta} + 0.4831 \sin \tilde{\delta} \\ 0.494 \cos \tilde{\alpha} \cos \tilde{\delta} - 0.445 \sin \tilde{\alpha} \cos \tilde{\delta} + 0.747 \sin \tilde{\delta} \end{pmatrix},\tag{19}$$

where  $\tilde{\alpha}_0 = 282.25^\circ$  is the right ascension of the equinox,  $\gamma \approx 33^\circ$  was given above and  $\theta_P = 62.6^\circ$  is the angle the Earth's north pole forms with the axis of the galaxy.

Due to the Earth's rotation the unit vector  $(x, y, z)$ , with a suitable choice of the initial time,  $\tilde{\alpha} - \tilde{\alpha}_0 = 2\pi(t/T)$ , is changing as a function of time

The angle  $\Theta$  scanned by the direction of observation is shown, for various inclinations  $\tilde{\delta}$ , in Fig. 3. We see that for negative inclinations, the angle  $\Theta$  can take values near  $\pi$ , i.e. opposite to the direction of the sun's velocity, where the rate attains its maximum.

The equipment scans different parts of the galactic sky, i.e. observes different angles  $\Theta$ . So the rate will change with time depending on whether the sense of the recoiling nucleus can be determined along the line of recoil. The results depend, of course, on the WIMP mass and the target employed. We will consider a light (the time dependence of  $\kappa$  is exhibited in Fig. 4) and an intermediate-heavy target (the time dependence of  $\kappa$  is exhibited in Fig. 5).

## 5 Discussion

In directional experiments one measures not only the energy of the recoiling nucleus but also the direction the nucleus is recoiling. Some of the requirements that should be met by such detectors have recently been discussed [16, 17]. To fully exploit the advantages of such detectors one should be able to distinguish between recoils with momenta  $\mathbf{p}$  and  $-\mathbf{p}$  (sense of direction), which now appears to be feasible. Some of the predicted interesting features of directional event rates persist, however, even if it turns out that the sense of motion of recoils along their line of motion cannot be measured. Such experiments, given a sufficient number of events, provide an excellent signature to discriminate against background. One expects, of course, a smaller rate by observing in a given direction. In the most favored direction, opposite to the sun's direction of motion, the event rate is  $\approx \frac{\kappa}{2\pi}$  down from that of the standard non directional experiments, if a specific angle  $\Phi$  is chosen. Since, however,  $\kappa$  is independent of the angle  $\Phi$ , one can integrate over all azimuthal angles and thus the retardation is only of order  $\kappa$  ( $\kappa \approx 1$  in the most favored direction). The parameter  $\kappa$  is essentially independent of any particle model parameters other than the WIMP mass. It does depend, however, on the assumed velocity distribution and to the nuclear form factor. Finally we have seen that in directional experiments the relative event rate for detecting WIMPs within our galaxy, as given by the parameter  $\kappa$ , will show a periodic diurnal variation due to the rotation of the Earth. The time variation is larger in the case of light WIMP and/or light target. So from this perspective the lighter target is preferred. The time variation arising after the inclusion of the modulation parameters  $(h_m, \alpha)$  or  $(h_c, h_s)$  is expected to be even more complicated, since these parameters depend on both  $\Theta$  and  $\Phi$ . One expects a diurnal variation on top of the annual variation characteristic of the usual modulation effect entering both directional and non directional experiments. Such effects will be discussed elsewhere.

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## References

- [1] S. Hanany *et al.*, *Astrophys. J.* **545**, L5 (2000); J.H.P Wu *et al.*, *Phys. Rev. Lett.* **87**, 251303 (2001); M.G. Santos *et al.*, *Phys. Rev. Lett.* **88**, 241302 (2002).
- [2] P. D. Mauskopf *et al.*, *Astrophys. J.* **536**, L59 (2002); S. Mosi *et al.* *Prog. Nuc.Part. Phys.* **48**, 243 (2002); S. B. Ruhl *et al.*, *astro-ph/0212229* and references therein.
- [3] N. W. Halverson *et al.*, *Astrophys. J.* **568**, 38 (2002); L. S. Sievers *et al.*, *astro-ph/0205287* and references therein.
- [4] G. F. Smoot *et al* (*COBE Collaboration*), *Astrophys. Journ.* **396**, L1 (1992).
- [5] A. H. Jaffe *Phys. Rev. Lett.* **86**, 3475 (2001).
- [6] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003).
- [7] D. Spergel *et al.*, *Astrophys. J.* **170**, 377 (2007); *arXiv:astro-ph/0603449v2*.
- [8] D. P. Bennett *et al.*, *Phys. Rev. Lett.* **74**, 2867 (1995).
- [9] M. W. Goodman and E. Witten, *Phys. Rev. D* **31**, 3059 (1985).
- [10] J. D. Vergados, *Phys. Rev. D* **57**, 103003 (2003); *hep-ph/0303231*.
- [11] J. D. Vergados, *J. Phys. G* **30**, 1127 (2004); *arXiv:hep-ph/0406134*
- [12] J. Vergados and A. Faessler, *Phys. Rev. D* **75**, 055007 (2007).
- [13] S. Ahlen, The case for a directional dark matter detector and the status of current experimental efforts, *cygnus2009Whitepaper*, Edited by J. B. R. Battat, *IJMPA* 25 (20010) 1; *arXiv:0911.0323 (astro-ph.CO)*.
- [14] J.D. Vergados and C.C. Moustakidis, *Cent. Eur. Jour. Phys*, to be published.
- [15] D. N. Spergel, *Phys. Rev. D* **37**, 1353 (1988).

- [16] C. J. Copi, L.M. Krauss, D. Simmons-Duffin. and S.R. Stroiney Phys. Rev. D **75**, 023514 (2007); arXiv:astro-ph/0508649.
- [17] A. M. Green and B. Morgan, Astropart. Phys. **27**, 142 (2007); arXiv:0707.1488 (astro-ph).